Section 1.4: Problem Solving "Something has to be x (or involve x)"

The 5-step plan on page 30 contains good ideas.

Normally, when we do an algebra problem, we first copy the original problem. We do NOT copy the word problem itself on our paper, however we do set up the word problem properly.

Each word problem has a single "Let" statement that defines each of the "participants" in the word problem in terms of a variable.

For example: "Three consecutive even numbers …. " would have a set up like this:

$$\begin{array}{llll} \text{Let} & x & = & \text{1st number} \\ & x+2 & = & \text{2nd number} \\ & x+4 & = & \text{3rd number} \end{array}$$

The next step is to translate the English word problem into one or more algebraic equations. Solve. Finally, you **must** write a complete sentence that responds to the original problem. You are **not** allowed to use abbreviations or pronouns.

Use proper mathematics style when working.

Beginning algebra students try to solve equations much like they write a sentence – that is from left to right across the page.

Algebra steps are written in a vertical manner as
$$\left\{\begin{array}{ccccc} y & - & 4.7 & = & 13.9 \\ & + & 4.7 & = & 4.7 \\ \hline y & & & = & 18.6 \end{array}\right.$$

- ⋆ If the original problem contains an equal sign, you are allowed ***only one = sign* per line**.
- ⋆ If the original problem does **not** contain an equal sign, you are **not** allowed **any** equal signs.

Work vertically! Notice there are **no equal signs** in the following.

$$\text{This: } \begin{cases} 2(6-2(3x-4)-5) \\ 2(6-6x+8-5) \\ 2(9-6x) \\ 18-12x \end{cases}$$

Instead of this: $2(6-2(3x-4)-5) \quad 2(6-6x+8-5) \quad 2(9-6x) \quad 18-12x$
or this: $2(6-2(3x-4)-5) = 2(6-6x+8-5) = 2(9-6x) = 18-12x$

A "Mathematical Model" is a function (a math equation) that gives values that match a physical situation.

$f(h) = 60h$ is a function that will give the number of minutes in h hours.

$$\left.\begin{array}{l} f(3) = 60 \cdot 3 \\ f(3) = 180 \end{array}\right\}$$ so in 3 hours there are 180 minutes.

Homework pages 39 - 41.

Remember the objective for these problems is to have a proper set-up of the problem.

Problem 1 should have the following set-up:

Let x = the smaller number
91 – x = the larger number

91 – x = x + 9

I normally like to "add to a smaller number" rather than subtract from a larger number. For this reason I added the 9 to the smaller number so that it would be equal to the larger number (per the story).

Another way to set up this problem is:

Let x = the smaller number
x + 9 = the larger number

x + x + 9 = 91

In this set up the thing to remember is that the small number plus the large number sums to 91.

It is important that you identify BOTH of the items in the story. Further, your equation **must be a direct translation** from the story.

It would have been incorrect to have written the equation 2x + 9 = 91 because this is not a direct translation of the story.

Page 38 problem 5

Let x = smallest angle
x + 1 = second angle
x + 2 = third angle

x + (x+1) + (x + 2) = 180

★ Notice **all three angles are defined**. The sum of the 3 angles of a triangle is 180 degrees.

Problem 9

We want to know the wholesale price so …

Let w = wholesale price

Translate the story into an algebraic equation: w + .50w + 1.50 = 22.50

Notice that we do not have dollar signs or percent symbols in our equation.

Problem 14

We have 3 angles. Which one is the smallest? I think the second angle is smallest so….

Let x = second angle
4x = first angle
2x + 5 = third angle

Notice how the third angle is defined. Five more than two times the "second angle". In fact, any place that "second angle" would appear in the story, you would substitute just the variable x.

In section 1.5, Page 41 and 42

Here the examples show some poor methods.

Examples 1 and 2 show "division" by drawing a line and putting the divisor below.

The examples should look like the following:

$$A = l \cdot w$$
$$\left(\frac{1}{w}\right)A = l \cdot w\left(\frac{1}{w}\right)$$
$$\frac{A}{w} = l$$

$$I = Prt$$
$$\left(\frac{1}{Pr}\right)I = Prt\left(\frac{1}{Pr}\right)$$
$$\frac{I}{Pr} = t$$

Notice that we do not "divide" but rather we multiply on both sides by what we need.

On page 42 in example 3, the text shows multiplying both sides by the necessary fraction.

Further down the example they show a multiplication that should have been first a reduce and then a multiplication.

$$\frac{2}{3}\cdot 9=\frac{2}{3}\cdot\frac{3}{2}(x+5)$$

$$\frac{2}{\cancel{3}}\cdot \overset{3}{\cancel{9}}=\frac{\cancel{2}}{\cancel{3}}\cdot\frac{\cancel{3}}{\cancel{2}}(x+5)$$

etc.

Example 4 is important to notice.

We want to solve for the letter P. P shows up in two terms on the right side. To make this problem work very nicely, we need to **"factor the common factor"** from those two terms. This might be easier to think of as how could I create an expression involving parentheses that would end up as $P+Prt$.

Would $P(rt)$ do the job? No.
Would $P(1+rt)$ do the job? Yes.

How are they different? The second one has a one so that P times the 1 is P *and* P times rt gives the Prt.

That is how they got to the second line of their example.

Section 1.6 Properties of exponents

Given: b^a . The b is the base and the a is the power (also known as the exponent).

$$\left(x^2\right)^3 \quad \rightarrow \quad x^{2\cdot 3}$$
$$\underbrace{x^2 \cdot x^2 \cdot x^2} \qquad x^6$$
$$x^{2+2+2}$$
$$x^6$$

$$\frac{x^5}{x^2} \quad \rightarrow \quad x^{5-2}$$
$$\frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x} \qquad x^3$$
$$\frac{\cancel{x} \cdot \cancel{x} \cdot x \cdot x \cdot x}{\cancel{x} \cdot \cancel{x}}$$
$$x^3$$

$$\frac{x^2}{x^5} \quad \rightarrow \quad x^{2-5}$$
$$\frac{x \cdot x}{x \cdot x \cdot x \cdot x \cdot x} \qquad x^{-3}$$
$$\frac{\cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x} \cdot x \cdot x \cdot x} \qquad \frac{1}{x^3}$$
$$\frac{1}{x^3}$$

This leads to a useful conclusion ….

$$\frac{x^4}{x^4} \quad \rightarrow \quad x^{4-4}$$

Consider: $\frac{x \cdot x \cdot x \cdot x}{x \cdot x \cdot x \cdot x}$ $\qquad x^0$ So x to the zero power is ONE!

$$\frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x}} \qquad 1$$
$$1$$

Note: x could not have been zero in the first place because we cannot have zero as a denominator.

Only items with identical bases can be simplified.

These cannot be simplified: $a^2 \cdot b^3$ $\qquad \dfrac{x^3}{y^5}$

This cannot be simplified without a change: $8 \cdot 2^4$. Since $8 = 2^3$ we could change the

expression $8 \cdot 2^4$ above to this and then we can simplify it:

$$2^3 \cdot 2^4$$
$$2^{3+4}$$
$$2^7$$

Negative exponents are source of confusion for many algebra students.

If we consider the basic rule for combining exponents when the bases are the same we can determine the role of negative exponents:

$$\frac{x^2}{x^4} \quad \rightarrow \quad x^{2-4}$$

$$\frac{x \cdot x}{x \cdot x \cdot x \cdot x} \qquad x^{-2}$$

$$\frac{\cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x} \cdot x \cdot x} \qquad x^{-2}$$

$$\frac{1}{x^2} \quad = \quad x^{-2}$$

And consider a negative in the denominator

$$\left\{ \begin{aligned} &\frac{x^2}{x^{-4}} \quad \rightarrow \quad x^{2-(-4)} \\ &\qquad\qquad x^{2+4} \\ &\qquad\qquad x^6 \end{aligned} \right.$$

Notice that in both examples, the term with the negative exponent "is moved to the other side of the fraction".

See page 54 example 6. Notice the x^{-2} and the y^{-5} are moved from the top to the bottom and the exponents are then positive. In a similar manner, the z and the w on the bottom, with negative exponents, are moved to the top then with positive exponents.

Example 7 makes an unnecessary reference to the product rule in part a and to the quotient rule in example b.

Page 54 at the bottom of the page and on page 55 just above example 8 illustrate how to clear parentheses that involve exponents.

$\left(2^3\right)^2$	$\left(2^3\right)^2$	$\left(2^3\right)^2$
$(2\cdot 2\cdot 2)^2$	$\left(2^3\right)\left(2^3\right)$	$2^{3\cdot 2}$
$(8)^2$	$8\cdot 8$	2^6
$8\cdot 8$		$2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2$
64	64	64

What we have here is a base (the two) is raised to a power (the 3) and that quantity is raised to a power (the 2).

The third column shows that we would multiply the exponents.

The same thing happens if a fraction is raised to a power. The sample in the lavender box should have looked more like this:

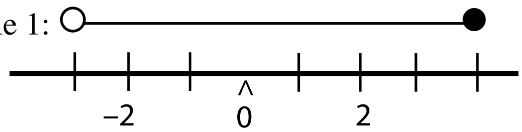
$$\left(\frac{a^b}{c^d}\right)^k = \frac{a^{bk}}{c^{dk}}$$

Notice that the exponent k is multiplied times the exponent b AND the k is also multiplied times the exponent d.

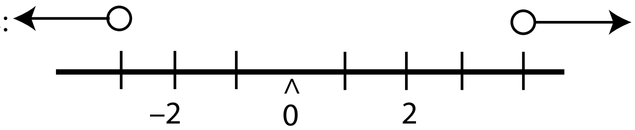
A summary of the properties of exponents is shown in the middle of page 57. You do not need to memorize this table! Just notice the similarities in the "rules". An important item to notice is that the numerator is a single term as is the denominator. That is to say, we do *not* have something that looks like this: $\frac{a+b^{-2}}{c^{-3}}$. The numerator is not a single term. It is a binomial. We will learn how to properly handle this situation in a later lesson.

Mr. Mumaugh
Math 104
Interval #2

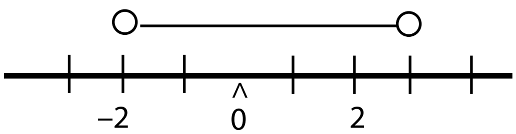
Find the statement for the intervals indicated on the number line in the group at the bottom of the page.

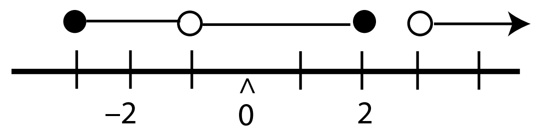
Example 1: 

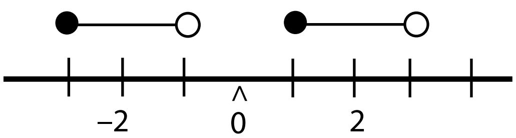
$-3 < x \leq 4$

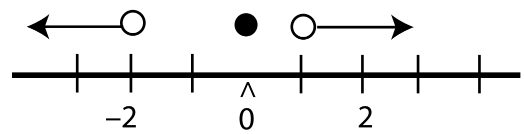
Example 2: 

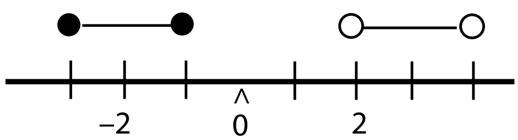
$x < -3$ or $x > 4$

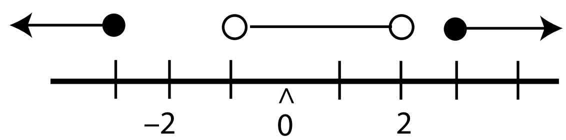
1) 

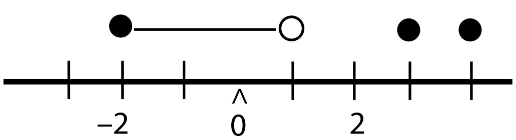
6) 

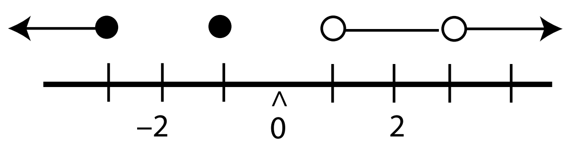
2) 

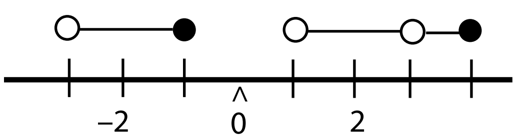
7) 

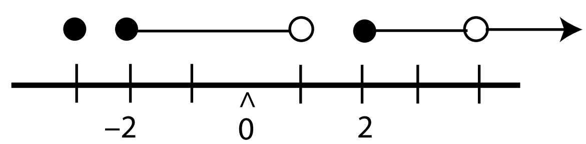
3) 

8) 

4) 

9) 

5) 

10) 

a) $-3 \leq x \leq 2$; $x \neq -1$ or $x > 3$

b) $-3 < x \leq -1$ or $1 < x \leq 4$; $x \neq 3$

c) $x = -3$ or $-2 \leq x < 1$ or $x \geq 2$; $x \neq 4$

d) $-2 < x < 3$

e) $x < -2$ or $x = 0$ or $x > 1$

f) $-3 \leq x \leq -1$ or $1 < x < 3$

g) $x < -2$ or $x = 0$ or $x > 1$

h) $-3 \leq x < -1$ or $1 \leq x < 3$

i) $x \leq -3$ or $-1 < x < 2$ or $x \geq 3$

j) $-2 \leq x < 1$ or $x = 3$ or $x = 4$